

We have learned about the coordinate plane, and that we can define points as pairs in that plane. A point a x-value of  $x$ , and y-value of  $y$ , would be...

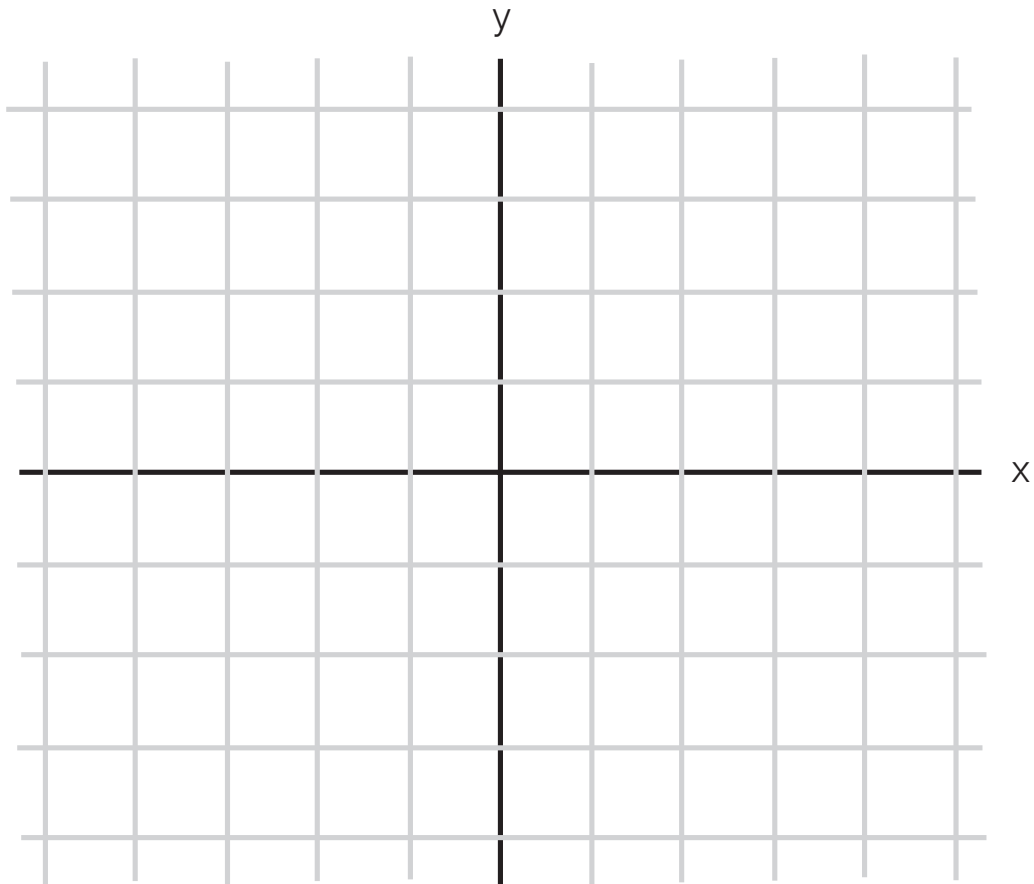
$$v = (x, y)$$

These point pairs are also called vectors. Vectors are useful, and you can add them and do all sorts of other useful operations with them.

$$v_1 = (x_1, y_1)$$

$$v_2 = (x_2, y_2)$$

$$v_1 + v_2 = (x_1 + x_2, y_1 + y_2)$$



Pick two vectors in x and y, add them together, and show the result.

But there's another way to think about a number in two dimensions.

Consider the number  $i$  that we've learned about...

$$i = \sqrt{-1}$$

We can also think about a number that has  $i$  in it as a 2-d number

$$c = x + iy$$

You can add these too!

$$c_1 = x_1 + iy_1$$

$$c_2 = x_2 + iy_2$$

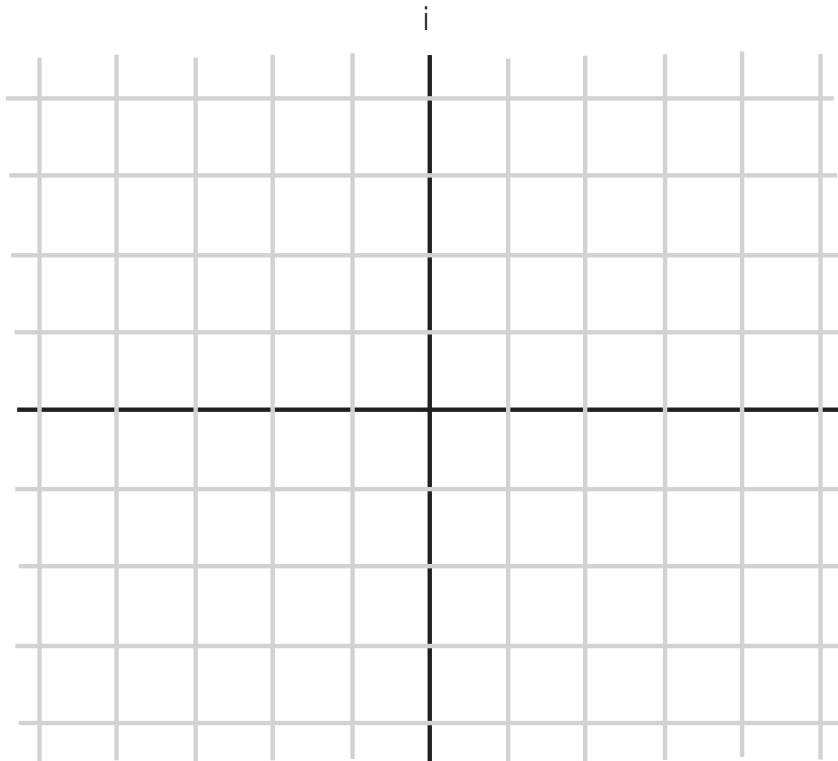
$$c_1 + c_2 = x_1 + x_2 + iy_1 + iy_2$$

Can you simplify the final sum into an equation with only 1 instance of  $i$ ?

$$c_1 + c_2 = (x_1 + x_2) + i($$

Look, no more ordered pairs!

Now, Draw two complex numbers and their sum, below...

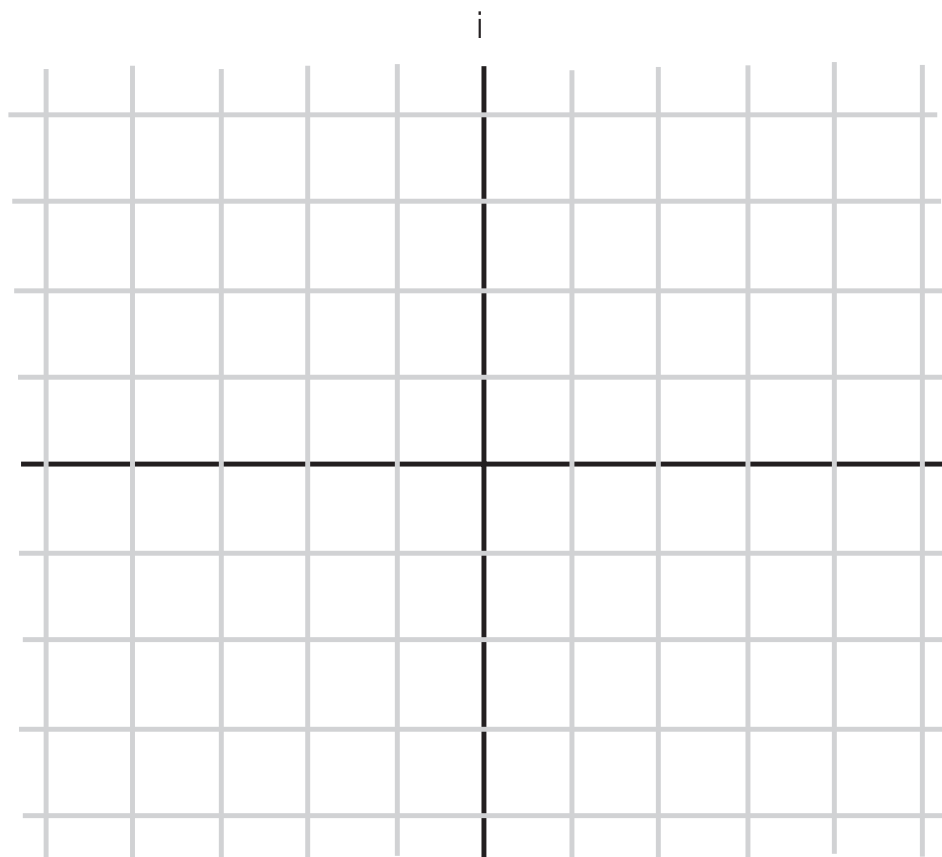


It's also possible to multiply complex numbers, just by knowing the definition of  $i = \sqrt{-1}$  and the normal rules of elementary algebra

Please simplify the product so that one instance of  $i$  shows up

$$\begin{aligned}c_1 &= x_1 + iy_1 \\c_2 &= x_2 + iy_2 \\c_1 * c_2 &= (x_1 + iy_1) * (x_2 + iy_2) \\&= x_1 * (x_2 + iy_2) + iy_1 * (x_2 + iy_2) \\&= \end{aligned}$$

Now, pick some complex numbers and draw their product



What do you notice about multiplying two complex numbers together when you plot the result?

Let's say you have a complex number of the form...

$$c_1 = x_1 + iy_1$$

How "long" is the line that goes from the origin (the middle of the coordinate axes) to the point  $c_1$ ?

If you could pick two complex numbers where the length of  $c_1$  and  $c_2$  was both 1, what would be the effect of multiplying them?

Now, pick some complex numbers of this kind and draw their product

